Solutions - Midterm Exam

(February 17th @ 7:30 pm)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (20 PTS)

- Compute the result of the following operations. The operands are signed fixed-point numbers. The result must be a signed fixed point number. For the division, use $x = 5$ fractional bits.

1.010001 + 1.011	1001.1101 − 1.011101	0.010101 + 01.11111
10.101 × 0.10011	1.011 × 1.0101	10.10010 ÷ 0.101

```
c8=1 c7=1 c6=0 c5=1 c4=0 c3=0 c2=0 c1=0 c0=0
 1 1.0 1 0 0 0 1 +
 1 1.0 1 1 0 0 0
-----------------
 1 0.1 0 1 0 0 1
```

```
1 0 0 1.1 1 0 1 0 0 -         c10=0 c9=0 c8=0 c7=1 c6=1 c5=0 c4=0 c3=0 c2=0 c1=0 c0=0
1 1 1 1.0 1 1 1 0 1     ➡     1 0 0 1.1 1 0 1 0 0 +
                              0 0 0 0.1 0 0 0 1 1
                              -------------------
                              1 0 1 0.0 1 0 1 1 1
```

```
c9=0 c8=0 c7=1 c6=1 c5=1 c4=1 c3=1 c2=0 c1=0 c0=0
0 0 0.0 1 0 1 0 1 +
0 0 1.1 1 1 1 1 0
-----------------
0 1 0.0 1 0 0 1 1
```

```
10.101  x  ➡  01.011  x  ➡    1 0 0 1 1 x
0.10011       0.10011            1 0 1 1
                              ---------
                              1 0 0 1 1
                            1 0 0 1 1
                          0 0 0 0 0
                        1 0 0 1 1
                      -----------------
                      0 1 1 0 1 0 0 0 1
                              ⬇
                      0.1 1 0 1 0 0 0 1
                              ⬇
                      1.0 0 1 0 1 1 1 1
```

```
1.011  x  ➡  0.101  x  ➡        1 0 1 1 x
1.0101       0.1011                1 0 1
                              -----------
                                1 0 1 1
                              0 0 0 0
                            1 0 1 1
                          -------------
                          0 0 1 1 0 1 1 1
                                ⬇
                          0.0 1 1 0 1 1 1
```

✓ $\frac{10.10010}{0.101}$: To unsigned (numerator) and then alignment, $a = 4$: $\frac{01.0111}{0.101} = \frac{01.0111}{00.1010} = \frac{010111}{001010} \equiv \frac{10111}{1010}$

```
        0001001001
1010 ) 1011100000
       1010
       ----
          1100
          1010
          ----
           10000
            1010
            ----
             110
```

Append $x = 5$ zeros: $\frac{1011100000}{1010}$

Integer Division:

$Q = 1001001, R = 110$

$\rightarrow Qf = 10.01001 \ (x = 5)$

Final result (2C): $\frac{01.01110}{1.011} = 2C(010.01001) = 101.10111$

PROBLEM 2 (10 PTS)

- Represent these numbers in Fixed Point Arithmetic (signed numbers). Use the FX format [12 4].
  - ✓ -16.375
    - +16.375 = 010000.011
    - ⇒ -16.375 = 11101111.1010
  - ✓ 32.3125
    - +32.3125 = 00100000.0101

- Complete the table for the following fixed point formats (signed numbers): (6 pts.)

Integer bits	Fractional Bits	FX Format	Range	Resolution
6	3	[9 3]	$[-2^5, 2^5 - 2^{-3}]$	2^{-3}
8	5	[13 5]	$[-2^7, 2^7 - 2^{-5}]$	2^{-5}

PROBLEM 3 (40 PTS)

- Perform the following 32-bit floating point operations. For fixed-point division, use 4 fractional bits. Truncate the result when required. Show your work: <u>how you got the significand and the biased exponent bits of the result</u>. Provide the 32-bit result.

✓ C1500000 + 436A0000	✓ D0A90000 – CF480000	✓ 80400000 × 7AB80000	✓ FBB80000 ÷ 49400000

✓ $X =$ C1500000 + 436A0000:

C1500000: 1100 0001 0101 0000 1000 0000 0000 0000

$e + bias = 10000010 = 130 \rightarrow e = 130 - 127 = 3$ $\quad Significand = 1.101$

C1500000 = -1.101×2^3

436A0000: 0100 0011 0110 1010 0000 0000 0000 0000

$e + bias = 10000110 = 134 \rightarrow e = 134 - 127 = 7$ $\quad Significand = 1.110101$

436A0000 = 1.110101×2^7

$X = -1.101 \times 2^3 + 1.110101 \times 2^7 = -\dfrac{1.101}{2^4} \times 2^7 + 1.110101 \times 2^7$

$X = (-0.0001101 + 1.110101) \times 2^7$

```
c9=1 c8=1 c7=1 c6=1 c5=0 c4=0 c3=0 c2=1 c1=0 c0=0
 0 1.1 1 0 1 0 1 0 +
 1 1.1 1 1 0 0 1 1
 -----------------
 0 1.1 0 1 1 1 0 1
```

To subtract these unsigned numbers, we first convert to 2C:

$R = 01.110101 - 0.0001101 = 01.110101 + 1.1110011$

The result in 2C is: $R = 01.1011101$

For floating point, we need to convert to sign-and-magnitude:

$\Rightarrow R(SM) = +1.1011101$

* You can also do unsigned subtraction: $X = (1.110101 - 0.0001101) \times 2^7$

```
b8=0 b7=0 b6=0 b5=1 b4=1 b3=1 b2=0 b1=1 b0=0
 1.1 1 0 1 0 1 0 -
 0.0 0 0 1 1 0 1
 ---------------
 1.1 0 1 1 1 0 1
```

$X = 1.1011101 \times 2^7$, $e + bias = 7 + 127 = 134 = 10000110$

X = 0100 0011 0101 1101 0000 0000 0000 0000 = 435D0000

✓ $X =$ D0A90000 – CF480000:

D0A90000: 1101 0000 1010 1001 0000 0000 0000 0000

$e + bias = 10100001 = 161 \rightarrow e = 161 - 127 = 34$ $\quad Significand = 1.0101001$

D0A90000 = -1.0101001×2^{34}

CF480000: 1100 1111 0100 1000 0000 0000 0000 0000

$e + bias = 10011110 = 158 \rightarrow e = 158 - 127 = 31$ $\quad Significand = 1.1001$

CF480000 = -1.1001×2^{31}

$X = -1.0101001 \times 2^{34} + 1.1001 \times 2^{31} = -1.0101001 \times 2^{34} + \dfrac{1.1001}{2^3} \times 2^{34}$

$X = -(1.0101001 - 0.0011001) \times 2^{34}$ (unsigned subtraction)

$X = -1.001 \times 2^{34}$, $e + bias = 34 + 127 = 161 = 10100001$

X = 1101 0000 1001 0000 0000 0000 0000 0000 = D0900000

```
b8=0 b7=0 b6=0 b5=1 b4=0 b3=0 b2=0 b1=0 b0=0
 1.0 1 0 1 0 0 1 -
 0.0 0 1 1 0 0 1
 ---------------
 1.0 0 1 0 0 0 0
```

✓ $X =$ 80400000 × 7AB80000:

80400000: 1000 0000 0100 0000 0000 0000 0000 0000

$e + bias = 00000000 = 0 \rightarrow Denormal\ number \rightarrow e = -126$ $\quad Significand = 0.1$

80400000 = -0.1×2^{-126}

7AB80000: 0111 1010 1011 1000 0000 0000 0000 0000

$e + bias = 11110101 = 245 \rightarrow e = 245 - 127 = 118$ $\quad Significand = 1.0111$

7AB80000 = 1.0111×2^{118}

$X = (-0.1 \times 2^{-126}) \times (1.0111 \times 2^{118}) = -0.10111 \times 2^{-8} = -1.0111 \times 2^{-9}$

$e + bias = -9 + 127 = 118 = 01110110$

X = 1011 1011 0011 1000 0000 0000 0000 0000 = BB380000

✓ $X =$ FBB80000 ÷ 49400000:

FBB80000: 1111 1011 1011 1000 0000 0000 0000 0000

$e + bias = 11110111 = 247 \rightarrow e = 247 - 127 = 120$ $\quad Significand = 1.0111$

FBB80000 = -1.0111×2^{120}

49400000: 0100 1001 0100 0000 0000 0000 0000 0000

$e + bias = 10010010 = 146 \rightarrow e = 146 - 127 = 19$ $\quad Significand = 1.1$

49400000 = 1.1×2^{19}

$$X = -\frac{1.0111 \times 2^{120}}{1.1 \times 2^{19}} = -\frac{1.0111}{1.1} \times 2^{101}$$

```
            000001111
      11000 101110000
             11000
             101100
              11000
              101000
               11000
               100000
                11000
                 1000
```

Alignment:

$$\frac{1.0111}{1.1} = \frac{1.0111}{1.1000} = \frac{10111}{11000}$$

Append $x = 4$ zeros: $\frac{101110000}{11000}$

Integer division

$$Q = 1111 \rightarrow Qf = 0.1111$$

Thus: $X = -0.1111 \times 2^{101} = -1.111 \times 2^{100}$

$e + bias = 100 + 127 = 227 = 11100011$

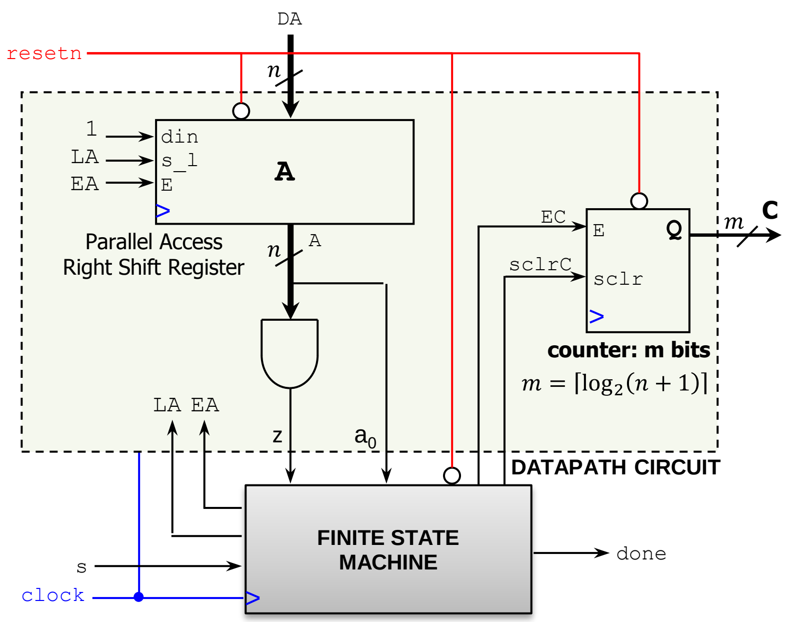
X = 1111 0001 1111 0000 0000 0000 0000 0000 = F1F00000

PROBLEM 4 (30 PTS)

- "Counting 0's" Circuit: It counts the number of bits in register A with a '0' value. The digital system is depicted below.
  - ✓ Example: for $n = 8$: if $A = 00110010$, then $C = 0101$.
  - ✓ The behavior (on the clock tick) of the generic components is as follows:

m-bit counter (modulo-n+1): If E=0, the count stays.	n-bit Parallel access shift register: If E=0, the output is kept.

`if E = 1 then`
`   if sclr = 1 then`
`      Q ← 0`
`   else`
`      Q ← Q+1`
`  end if;`
`end if;`
	`if E = 1 then`
	`   if s_l = '1' then`
	`      Q ← D`
	`   else`
	`      Q ← shift in 'din'` (to the right)
	`  end if;`
	`end if;`

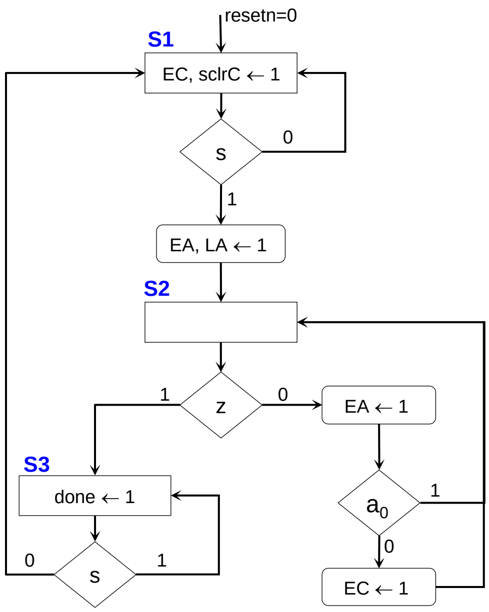


ALGORITHM

```
C ← 0
while A ≠ 11...1 (2ⁿ-1)
   if a0 = 0 then
      C ← C + 1
   end if
   right shift A
end while
```

- Sketch the Finite State Machine diagram (in ASM form) given the algorithm (for n=8, m=4). (18 pts.)
  - ✓ The process begins when s is asserted, at this moment we capture DA on register A. Then, we shift A one bit at a time. The process ends when $A = 2^n - 1$ (i.e., when z=1). The signal done is asserted when we finish counting.
  - ✓ As A is being shifted: we need to increase the count C every time $a_0 = 0$.

Finite State Machine:



- Complete the timing diagram (n=8, m=4). A is represented in hexadecimal format, while C is in binary format (12 pts.)